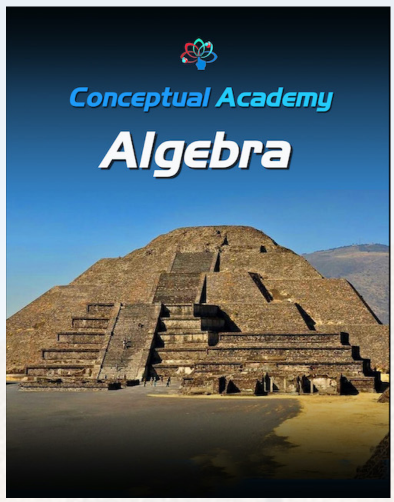
Conceptual Math
Algebra I

Chapter 9: Linear Equation Applications



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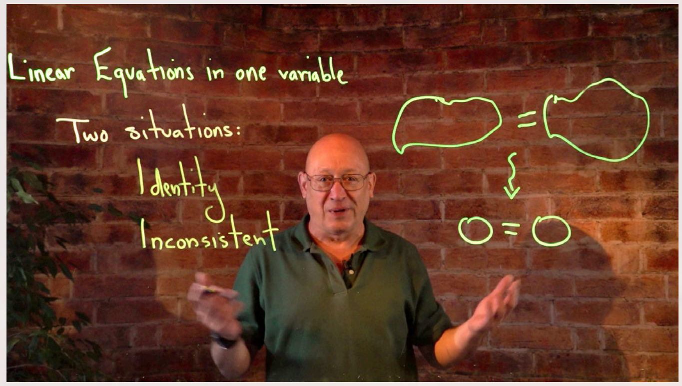

Chapter 9

Linear Equation Applications

9.1 Related Quantities

The Relationship Problem – Situations can arise where different quantities are related to each other in a certain way, and they also have to satisfy a condition. By letting a variable represent one of the quantities, we can produce an equation whose solution identifies all of the values involved.

Their relationships are specified, and together they meet a condition. We let a variable, say x, be ONE of the quantities. The others can then be expressed in terms of x. The condition becomes an equation.

EXAMPLE: Bob is 10 years older than Sally. Sally is twice as old as Stevie. Their ages add to 60. How old are they?

The quantities are Bob's age and Sally's age. They are related by being ten years apart. The condition they must satisfy is adding to 60.

Let $x =$ Sally's age. Since Bob is ten years older, his age is $x + 10$.

The ages add to 60, so we have:

Name	Age
Bob	$x + 10$
Sally	x
Total	60

From the second column we see that x + 10 + x = 60.

2x = 50

x = 25

Sally is 25 and Bob is 35.

WARNING – Since there are multiple quantities in the problem, beginners are tempted to use more than one variable. DON'T DO THIS!!

EXAMPLE:
A jar contains twice as many blue marbles as yellow marbles, and 20 more green marbles than yellow marbles. If the total number of marbles in the jar is 80, how many are there of each color?

HELPFUL PRACTICE: Read the problem once for an overview of the situation. Read it again and write down each quantity identified. Read it again and identify the condition they must satisfy.

Choose a quantity to be the variable and express the values of the other quantities in terms of that variable. Do not introduce another variable.

Write the equation that expresses the condition. Solve the equation.
We have blue, yellow, and green marbles. List them and let x = the number of yellow marbles.:

There are total of 80 marbles.

Color	Count
Blue	2x
Yellow	x
Green	x + 20
Total	80

2x + x + x + 20 = 80

4x = 60

x = 15

So we know that there are 15 yellow marbles.

This means there are 30 blue marbles and 35 green marbles. Do these add to 80?

15 + 30 + 35 = 80? YES.

NOTE: With these problems (and really, any word or application problem) it is important to be clear regarding the question being asked. The value of the variable used might not represent the quantity requested in the question.

EXAMPLE: A taco truck sells bottles of Pepsi, Coke, Dr. Pepper, and Sprite. Prior to leaving its station, the truck has twice as many Pepsi as Sprite and 20 more Coke than Pepsi. It has 10 more Sprite than Dr. Pepper. It has 180 bottles. How many are Pepsi?

Let x = the number of bottles of Sprite.

Soda	Number of bottles
Sprite	x
Pepsi	2x
Coke	2x + 10
Dr. Pepper	x - 10
Total	180

We get:

x + 2x + 2x + 10 + x - 10 = 180

6x = 180

x = 30

If we answer the problem with 30 bottles, we are WRONG. That is the number for Sprite. The problem asked for Pepsi. The correct answer is 60. If it had asked for Coke, the answer is 70 bottles.

9.2 Motion

Even before we learn to drive, we become familiar with the idea of how long it takes to travel a certain distance when we move at a given speed. We know that if we're driving a number of miles per hour, then the time required for a distance is to divide it by our speed. This is expressed by the equation

Distance = (Rate)(Time) or D = RT.

Obviously we have to put these numbers in the right measurements for the equation to make sense. We can't express the distance in kilometers and the speed in miles per hour.

When a single obiect is moving at a constant speed, simple arithmetic can calculate the quantities involved, but when there are multiple objects, or different speeds traveled at different times, we require algebra to express the relationships.

EXAMPLE: Bob leaves the house and drives 50 miles per hour. One hour later, Sally leaves and chases Bob driving 70 miles per hour. How long does it take Sally to catch Bob?

SOLUTION: Write D=RT equations for Bob and Sally.

Bob: $D_B = 50T_B$ Sally: $D_S = 70T_S$

Here we arrive at what is perhaps the single most critical concept in all of algebra: the ability to work with values while they remain unknown. We don't know Bob or Sally's time when she catches him, but we know THE RELATIONSHIP. We know that Bob has driven one hour longer.

We don't know how far Bob or Sally have driven when she catches him, but we know IT IS THE SAME.

We know $D_B = D_S$

Which means $R_B T_B = R_S T_S$

Or $50T_B = 70T_S$

Since Bob drove one hour longer, write $T_B = T_S + 1$:

$50(T_S + 1) = 70T_S$

$50T_S + 50 = 70T_S$

$50 = 20T_S$

$T_S = 2.5$

Sally drove 2.5 hours, so Bob drove 3.5 hours. Using their speeds we can see that they drove 175 miles at the point she catches him.

EXAMPLE: A military aircraft departs on a mission and flies 400 mph towards its destination. The situations changes and it becomes necessary to dispatch a second aircraft to catch the first. The second plane leaves 90 minutes later and flies at 600 mph. How far did it have to fly to catch the first plane?

This is the same problem.

We know $D_1 = D_2$

Which means $R_1 T_1 = R_2 T_2$

Or $400T_1 = 600T_2$

Watch units! Express time in hours. 90 minutes = 1.5 hours.

$400(T_2 + 1.5) = 600T_2$

$400T_2 + 600 = 600T_2$

$600 = 200T_2$

$T_2 = 3$. The second plane flew for 3 hours to catch the first plane.

Suggestion: Express the time of the second plane as T_1 - 1.5 and confirm that the result is the same.

These problems capture the essence of the algebra this course is presenting. We don't know certain values, but we know RELATIONSHIPS. We use the relationships to produce an equation we can solve.

Did we answer the questions that was asked?

No. It asked for the distance the planes traveled. We can use the D=RT for either plane. Using the second plane, we have D = (600)(3) = 1800 miles. The plane flew 1800 miles.

EXAMPLE: Flight 324 leaves Los Angeles for New York at 11 AM. At the same time, Flight 1105 leaves New York for Los Angeles. If 324 flies at 450 mph and 1105 flies at 525 mph, and the distance from Los Angeles to New York is 2,450 miles, how many miles from each city are they when the two planes fly past each other?

Plane leaving Los Angeles:	D_L	$R_L T_L$	$450T_L$
Plane leaving New York:	D_N	$R_N T_N$	$525T_L$

Now we consider the relationships. They MUST be there. Read the problem as many times as necessary to note the relationships. The planes depart at the same time, so their travel times are equal. What can we say about the distances?

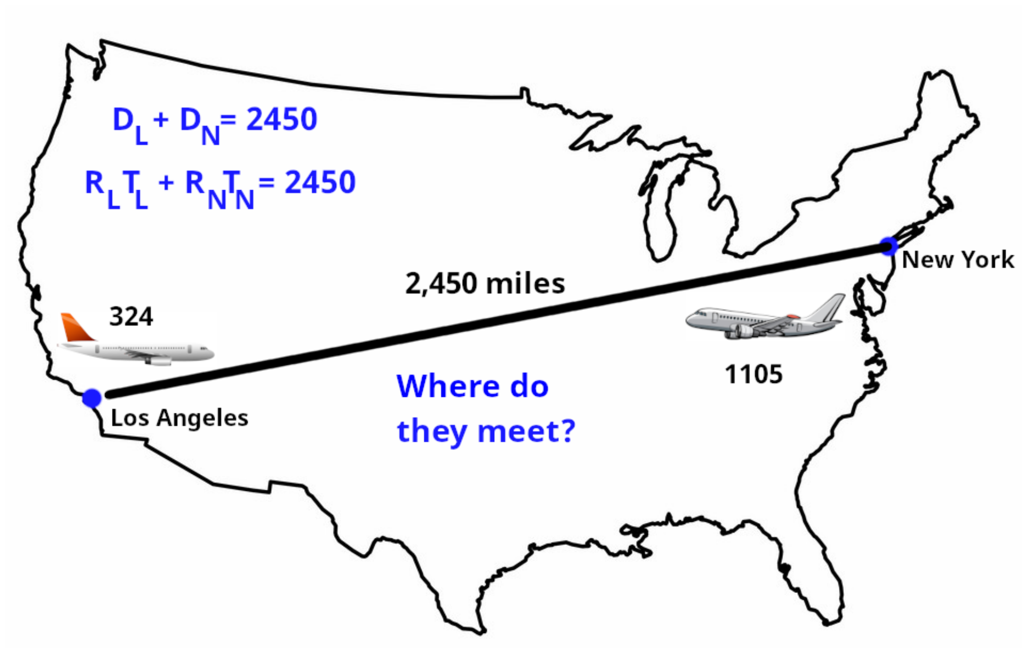

Figure 9.1: Flights 324 and 1150

They must add to 2,450 miles. We have $D_L + D_N = 2{,}450$ miles, or

$R_L T_L + R_N T_N = 450 T_L + 525 T_N = 2450$

and $T_L = T_N$ so just call it T.

450T + 525T = 2450.

975T = 2450.

T = 2.5 hours.

What question did the problem ask? The distance from LA is (450)(2.5) and the distance from NY is (525)(2.5). The planes were 1129 miles from LA and 1321 miles from New York.

9.3 Mixtures

Mixture applications involve combining or blending quantities of materials with a common property but at different levels.

It could be mixing:

• two blends of coffees with different prices per pound.

• two salt solutions with different percentages of salt dissolved

• two paint mixtures with different portions of paint colors contained in their blend

A 10% saltwater solution is 10% salt and 90% water.

KEY OBSERVATION: The amount of salt in a solution is equal to the salt percentage of the solution times the volume of the solution.

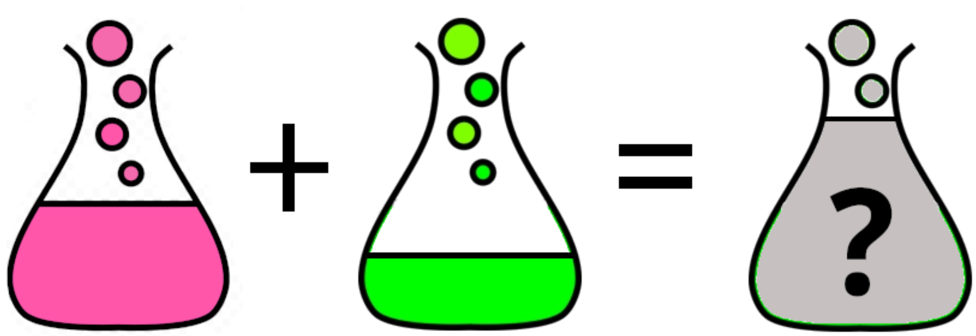

Figure 9.2: Combining Mixtures

Example: What is the percentage of salt in a mixture that is made by combining 2 liters of 20% salt solution with 1 liter of 60% salt solution?

Solution	Conc	Vol	(C)(V)	Result
One	20 %	1.0	(0.20)(1)	0.2
Two	60 %	2.0	(0.60)(2)	1.2
Blend	x %	3.0	3x	1.4

The 1.4 liters of salt is calculated as the sum of the products above it, and we know that this must also be the product of 3x, which gives us x = 0.47 or a 47% salt solution.

Example: How many pounds of a coffee blend that is $7.50 / pound should be mixed with a coffee that is $ 12 / pound to produce 5 pounds of coffee that is worth $ 9 / pound?

Let x = the pounds we require of $ 7.50 per pound coffee

We know that we must add to 5 pounds, so this means we require 5 – x of the $ 12 / pound coffee.

Coffee	Price / lb	Weight	(P)(W)	Cost
$ 7.5	7.5	x	7.5x	7.5x
$ 12	12	5 - x	12(5-x)	60 - 12x
$ 9	9	5.0	9(5)	45

We get the equation:

$(7.50)\ (x) + (12)\ (5 - x) = 45$

$7.5\ x + 60 - 12x = 45$

$7.5x - 12x = -\ 15$

$-\ 4.5x = -\ 15$

Or x = 3.33 or 3 $\frac{1}{3}$ pounds

So we have 3 $\frac{1}{3}$ pounds of coffee worth $ 7.50 / pound and 1 $\frac{2}{3}$ pounds of coffee worth $ 12 / pound. This will produce 5 pounds of coffee at $ 9 / lb.

9.4 Work

The work application involves the time required to complete a given task when multiple items (people, machines...) work together. We produce the equation for this by recognizing that if it takes a person 5 hours to complete a job, then after one hour this person has completed $\frac{1}{5}$ of the job. After $t\ hours$, $\frac{t}{5}$ of the job is completed.

This gives us an equation for the portion of the job completed after working t hours. If it takes a machine M hours to perform a task, then the portion completed after t hours is $\frac{t}{M}$. When that portion is 100% (or 1), the job is complete.

This is not interesting for one person or one machine, but when multiple items are working, the equation remains simple and allows us to find the time required when they work together.

Bob requires 3 hours to clean the gutters. Sally can do it in 2 hours. How long would it take for them to clean the gutters if they work together?

If we let t = the time worked in hours, then we know that Bob completes $\frac{t}{3}$ of the job in t hours and that Sally completes $\frac{t}{2}$ of the job in t hours.

To complete the job once, we have $\frac{t}{3} + \frac{t}{2} = 1$.

$\frac{2t}{6} + \frac{3t}{6} = 1$

$\frac{5t}{6} = 1$

5t = 6

We get t = 6/5 or 1.2 hours to complete the job if they work together.

To properly disinfect a restaurant's kitchen each night, a team of three cleaners work together after the restaurant closes. Working alone, Cindy requires 5 hours, Bob needs 8 hours, and Dave can finish in 6 hours. How much time is necessary to disinfect the kitchen if they work together?

The task is complete when the portion completed is 1, so we have:

$$\frac{t}{5} + \frac{t}{8} + \frac{t}{6} = 1.$$

$$\frac{24t}{120} + \frac{15t}{120} + \frac{20t}{120} = 1.$$

$$\frac{59t}{120} = 1.$$

$$t = \frac{120}{59} \text{ hours}.$$

It takes them almost exactly 2 hours to disinfect the kitchen.